

Dualities in Five Dimensions and Charged String Solutions

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Abstract

We consider an eleven dimensional supergravity compactified on $K3 \times T^2$ and show that the resulting five dimensional theory has identical massless states as that of heterotic string compactified on a specific five torus T^5 . The strong-weak coupling duality of the five dimensional theory is argued to represent a ten dimensional Type *IIA* string compactified on $K3 \times S^1$, supporting the conjecture of string-string duality in six dimensions. In this perspective, we present magnetically charged solution of the low energy heterotic string effective action in five dimensions with a charge defined on a three sphere S^3 due to the two form potential. We use the Poincare duality to replace the antisymmetric two form with a gauge field in the effective action and obtain a string solution with charge on a two sphere S^2 instead of that on a three sphere S^3 in the five dimensional spacetime. We note that the string-particle duality is accompanied by a change of topology from S^3 to S^2 and viceversa.

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I. INTRODUCTION

Our understanding of string theory is strongly guided by its rich symmetry contents. Among various symmetries, the target space symmetries such as T -duality and S -duality have been at the center of attention in the recent years. It is noted that the former can be tested in the perturbative frame work, interchanging the Kaluza-Klein modes with the string winding modes, whereas the latter is a nonperturbative phenomena and relates the strong and weak coupling regimes.

In this context, we briefly review some of the essential evidences related to various dualities. There have been some progress to understand the nonperturbative aspects of string theory [1] by studying the string-fivebrane duality [2] in ten dimensions. It has been argued that the fundamental string and the solitonic string are two different formulations of a single theory with strongly coupled string corresponding to weakly coupled fivebrane and viceversa [3]. For example, if e^{ϕ_0} is the string coupling (ϕ_0 being the dilaton) then the corresponding fivebrane coupling is $e^{-\phi_0/3}$. A nice review can be found in ref. [4].

Recently, the dynamics of string in various dimensions have been discussed by Witten [5], suggesting a series of interconnections between different string theories. There are evidences [6] that the six dimensional heterotic string derived from the ten dimensional one after compactification on T^4 , is dual to Type IIA string in six dimensions which is obtained from its ten dimensional counterpart by compactification on $K3$. As a result, the spectrum of Bogomol'nyi saturated states in the heterotic string theory on $M^6 \times T^4$ is identical to that of Type IIA string on $M^6 \times K3$ [7]. It has been shown that the string-string duality in six dimensions relates the fundamental heterotic string singular solutions to the solitonic heterotic string [8]. In this context, it has been conjectured that an eleven dimensional supergravity theory compactified on $K3$ can be interpreted as a heterotic string theory on a three torus following string-membrane duality [9] in seven dimensions. At present with the exisiting duality conjectures, it is argued that the eleven dimensional supergravity theory with an underlying low energy membrane theory is a plausible candidate for the origin of various string theories [10]. In this connection, one of the authors has considered [11] the Type IIA and Type IIB string theories in eight dimensions and has shown that the resulting p-branes can be classified in the $SL(3, Z) \times SL(2, Z)$ representaion. The strong reason behind all the speculations is the fact that the resulting theories have identical massless moduli spaces of vacua and the low energy spectrum along with same supersymmetry.

Now, let us recall the idea of duality between electric and magnetic charges in four dimensions [12], in connection to the monopole solutions [13] to Yang-Mills-Higgs theory. Subsequently, the electric-magnetic duality under the name S -duality have been discussed

in $N = 4$ supersymmetric Yang-Mills theory [14]. However, at present there are increasing evidences for the S -duality symmetries relating the strong and weak coupling regimes in the $N = 2$ supersymmetric gauge theories [15] and in $N = 1$ supersymmetric theories [16]. In sequel, the duality symmetry in string theories [17] have been studied in great detail in order to understand the string theory nonperturbatively [18]. The stringy realization of the results of $N = 2$ Yang-Mills theory [15] have been discussed in literature [19]. It has been shown that the duality transformation exchanges the electrically charged elementary string excitations with the magnetically charged soliton states [20] in the Narain's toroidal compactification [21] of heterotic string theory. There are also attempts to describe fundamental string as solitons in the dual string theory by utilizing various analogies between the superstrings with the solitons in the supersymmetric theories [22]. These string solitons can be constructed from the fivebrane solitons [23] with four of their world volume dimensions wrapped in the internal directions. In ref. [24] instanton and brane solutions are discussed in Type IIB string theory. Some of these solitons have been interpreted as the monopole solutions [25]. It has been shown that they saturate the Bogomol'nyi bound preserving half of the supersymmetries [26]. These BPS saturated states may play an important role in the nonperturbative dynamics of string theory. It has been argued that the study of such configurations may shed light on duality between strong-weak coupled string vacua.

Furthermore, in a series of papers [5,27–33], various predications of the duality conjectures are tested for the heterotic string theory along with Type IIA and IIB string theories. One of the most exciting result is that the strong coupling limit of Type IIA string in ten dimensions is an eleven dimensional supergravity [5,34], representing an underlying low energy membrane theory [35]. Since a membrane is T -dual to a fivebrane theory in eleven dimensions, the string-fivebrane duality in ten dimensions can be interpreted as a result of double dimensional reduction on S^1 of the corresponding membrane-fivebrane duality in eleven dimensions [36]. However, the membrane and fivebrane theories have not been understood completely as the quantum corrections are yet to be discovered. Note that, whenever we refer to the eleven dimensional supergravity theory, the underlying low energy membrane theory of world volume is understood.

In this paper, we investigate the dynamics in five dimensions corresponding to dimensionally reduced membrane theory, heterotic string, Type IIA and Type IIB string theories with $N = 2$ supersymmetry. We seek classical solutions of the heterotic string effective action with a motivation to provide evidence for various dualities. Note that, in five dimensions the two form potential is Poincare dual to the gauge field and the elementary string states are not charged in the spectrum with respect to this new gauge field. It has already been argued that such an electric charge corresponding to the new gauge field may arise in order

to satisfy the anomaly equation involving three rank antisymmetric field strength and gauge field strength [5]. We show that the string-particle duality in five dimensions leads to various theories admitting classical solutions with charge defined on different topologies, *e.g.* S^3 and S^2 etc. We present a five dimensional solitonic magnetically charged string solutions of the heterotic string effective action and apply Poincare duality to obtain another charged solution with charge defined on a different topology than the one before. The important point specific to five dimensions is the fact that the mass of charged *BPS* states is inversely proportional to the square of the coupling constant of the theory. Thus, in the strong coupling limit the mass of the multiplets tend to zero which is very unlikely in order to interpret the mass spectrum in five dimensions. Similarly, in the weak coupling limit, the mass of the charged states diverges. Thus the *BPS* value for the mass of the charged states leads to difficulty in interpreting the mass spectrum in the strong and weak coupling regimes. However it is argued that these charged states can be interpreted as Kaluza-Klein states on $R^5 \times S^1$ in order to overcome the puzzle in five dimensions [5]. The *T*-duality does not act on the new compactified coordinate represented by a circle S^1 . The above analysis in five dimensional theories has pointed out the need for a six dimensional theory (as the radius of S^1 is very large) in the strong coupling regime. As a result, the uncompactified theory seems to have eleven dimensions instead of ten dimensions. Thus it is natural to start with an eleven dimensional supergravity theory [37] which has an underlying low energy membrane interpretation.

Now, we propose to explore the duality symmetry between ten dimensional heterotic string theory compactified on a five dimensional torus T^5 , a Type *IIA* string on $K3 \times S^1$, Type *IIB* string on $K3 \times S^1$ and an eleven dimensional $N = 1$ supergravity theory compactified on $K3 \times T^2$. The bosonic part of the ten dimensional heterotic string action contains massless fields such as dilaton, graviton, antisymmetric tensor field and 16 abelian gauge fields. We show that the resulting five dimensional heterotic string theory has 27 gauge bosons; 26 of which transform as vectors under the global noncompact group $O(5, 21)$ and one comes after Poincare dualizing the field strength of antisymmetric tensor. On the other hand, compactification of the eleven dimensional $N = 1$ supergravity theory on $K3 \times T^2$ has $N = 2$ spacetime supersymmetry and describes a five dimensional low energy string effective action. At generic points in the moduli space of $K3 \times T^2$, the gauge symmetry of $D = 11$ supergravity theory is enhanced similar to the case of Type *IIB* superstring compactified on the $K3 \times S^1$ [5] and Type *IIA* string compactified on $K3$ [38]. It has been argued [5] that for a Type *IIB* string compactified on $K3$, the *U*-duality group is $SO(21, 5; \mathbb{Z})$; which is a combination of the corresponding *T*-duality group $SO(20, 4; \mathbb{Z})$ and the $SL(2, \mathbb{Z})$ symmetry of the ten dimensional Type *IIB* string theory. In fact the moduli

space of vacua of Type *IIB* theory compactified on $K3$ is identical to that of a heterotic string theory in five dimensions. However there are 5 self dual two forms, 21 antiself dual two forms in contrast to the 26 gauge fields in case of the heterotic string theory in five dimensions. Further compactification of the six dimensional Type *IIB* string on a circle S^1 gives rise to 26 gauge fields with enhanced gauge symmetry at the generic points in the moduli space and can be identified with a heterotic string compactified on a five torus.

We outline the paper as follows. In section II, we present a low energy heterotic string effective action in five dimensions obtained by toroidal compactification of the ten dimensional one and discuss the background field contents. The Poincare duality in $D = 5$ is used to construct the dual form of the effective action. In section III, we deal with the eleven dimensional supergravity which can be considered as the low energy limit of a membrane. We write down the $D = 11$ supergravity theory compactified on $K3$ and further compactify it on a two torus to obtain a five dimensional theory. In fact we use double dimensional reduction on $K3$ to arrive at the underlying worldsheet picture in seven dimensions from the corresponding worldvolume describing a membrane in eleven dimensions. In the next step, we compactify the spacetime on a two torus T^2 and arrive at a five dimensional theory possessing string-particle duality from the original eleven dimensional membrane-fivebrane duality. With field redefinitions along with the Poincare duality on the three form potential, we show in section IV that the number of moduli fields and the gauge field multiplets are identical to that of a toroidally compactified heterotic string in $D = 5$. The strong-weak coupling limit of eleven dimensional supergravity theory compactified on $K3 \times T^2$ is discussed in section V and the massless spectrum is identified with that of Type *IIA* string compactified on a $K3 \times S^1$, supporting the conjecture in six dimensions. In section VI, we present solitonic solution of the heterotic string effective action with charge defined on a three sphere S^3 due to the two form potential. We analyze another magnetically charged solution with a S^2 geometry of the dual heterotic string effective action and discuss the consequences of the Poincare and strong-weak couple dualities under the name string-particle duality in five dimensions. Finally in section VII, we summarize our result with various interconnections between different string theories in five dimensions along with the classical charged solutions.

II. HETEROTIC STRING EFFECTIVE ACTION IN FIVE DIMENSIONS

We begin this section by toroidally compactifying a $D = 10$ heterotic string theory to a five dimensional one. The resulting theory has $N = 2$ supersymmetry and contains five Kaluza-Klein gauge fields and five winding gauge fields from the ten dimensional metric

and antisymmetric tensor respectively. The elementary string states are electrically charged with respect to these gauge fields. In addition to the ten gauge fields there is also one more spin one field which is indeed Poincare dual to the antisymmetric tensor in five dimensions. However the state corresponding to the new gauge field is not necessarily electrically charged because of its appearance specific to five dimensions. The five dimensional effective action at generic points in the moduli space is manifestly invariant under the $O(5, 21; Z)$ transformations.

Now we start by writing down the bosonic part of the low energy heterotic string effective action compactified on a five torus T^5 following the general prescription of ref. [39] as

$$S_h = \int d^5x \sqrt{-g} e^{-\Phi} \left[R + g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{12} g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'} H_{\mu\nu\rho} H_{\mu'\nu'\rho'} \right. \\ \left. - \frac{1}{4} g^{\mu\mu'} g^{\nu\nu'} F_{\mu\nu}^{(a)} (\mathcal{L} \mathcal{M} \mathcal{L})_{ab} F_{\mu'\nu'}^{(b)} + \frac{1}{8} g^{\mu\nu} \text{Tr}(\partial_\mu \mathcal{M} \mathcal{L} \partial_\nu \mathcal{M} \mathcal{L}) \right], \quad (1)$$

where the three form antisymmetric field strength is obtained from the ten dimensional counterpart by toroidal compactification and is defined with a gauge Chern-Simon term as

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} - \frac{1}{2} A_{[\mu}^{(a)} \mathcal{L}_{ab} F_{\nu\rho]}^{(b)}$$

and the two form gauge field strengths are given by

$$F^{(a)}_{\mu\nu} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)}. \quad (2)$$

The three form H satisfies the modified Bianchi identity involving the gauge field strengths $F^{(a)}$ [5]. The massless bosonic background field configuration ($0 \leq \mu \leq 4$) include the metric $g_{\mu\nu}$, the antisymmetric tensor $B_{\mu\nu}$, the abelian gauge fields $A_\mu^{(a)}$ for $a = 1, 2 \dots 26$, the dilaton Φ and a (26×26) matrix valued scalar field \mathcal{M} representing an element of the coset given by

$$\mathcal{M} \in \frac{O(5, 21)}{O(5) \times O(21)}, \quad (3)$$

where $\mathcal{M}^T = \mathcal{M}$ and $\mathcal{M} \mathcal{L} \mathcal{M}^T = \mathcal{L}$, for $\mathcal{L} = \begin{pmatrix} -I_5 & 0 \\ 0 & I_{21} \end{pmatrix}$.

The moduli field \mathcal{M} is constructed from the internal components of a set of ten dimensional fields namely; the metric, the antisymmetric tensor field and the gauge fields. There are 15 scalars originating from the ten dimensional metric, 10 of them from ten dimensional antisymmetric tensor and the remaining 80 of them from the gauge fields in ten dimensions. Altogether, there are 106 scalar fields out of which 105 of them (φ^i) arise from the parametrization of the coset space (3) in terms of target space coordinates, $i = 1, 2, \dots 105$

and the remaining one represents a dilaton Φ . There are altogether 26 gauge fields, out of which 5 each originate from the ten dimensional metric and antisymmetric tensor respectively. The remaining 16 gauge fields may be identified with the diagonal generators of $SO(32)$ or $E_8 \times E_8$.

For later convenience, we introduce the matrix M_{ij} in the coset space (3) and rewrite the effective action (1) in the generic form following the convention of Townsend [9],

$$S_h = \int d^5x \sqrt{-g} e^{-\Phi} [R + (\partial\Phi)^2 - \frac{1}{12}H^2 - \frac{1}{4}F^{(a)}C_{ab}F^{(b)} - M_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j], \quad (4)$$

where C_{ab} is a positive definite matrix related to M_{ij} by a matrix transformations and is a function of the scalars φ^i . The explicit form of the matrix C_{ab} and M_{ij} can be calculated for the toroidal compactification.

Now, let us invoke the Poincare string-particle duality in five dimensions relating the three rank field strength to its dual *i.e.* a two rank field strength. We write the duality transformation as

$$e^{-\Phi} \partial^{[\mu} B^{\nu\rho]} = \frac{1}{2!\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma\lambda} \tilde{F}_{\sigma\lambda}, \quad (5)$$

where \tilde{F} is a Poincare dual of the three form field strength H which is only specific to five dimensions. Rewriting the the action in eq.(4) in terms of the gauge field strengths only, we arrive at

$$S_h = \int d^5x \sqrt{-g} e^{-\Phi} \left[R + (\partial\Phi)^2 - \frac{1}{4}e^{2\Phi}\tilde{F}^2 - \frac{1}{4}F^{(a)}C_{ab}F^{(b)} - M_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j \right] + \frac{1}{4} \int A^{(a)} \wedge \mathcal{L}_{ab} F^{(b)} \wedge \tilde{F}_{\sigma\lambda}, \quad (6)$$

with a topological term. The last term arises out of the gauge Chern-Simon term in the definition of three form H as defined in eq.(2). Note that, there are altogether 27 gauge fields in eq.(6); out of which the one corresponding to the dual of the two form potential appears with a different string coupling from the remaining 26 of them. The action in eq.(6) can be given an underlying particle interpretation, as we proceed, by identifying with a supergravity action in five dimensions.

III. ELEVEN DIMENSIONAL SUPERGRAVITY COMPACTIFIED ON $K3 \times T^2$

In this section, we present a five dimensional theory by dimensional reduction of $N = 1$ eleven dimensional supergravity [37]. We seek for the $D = 5$ theory from the present perspective of duality where it has been conjectured that the strong coupling limit of a ten

dimensional Type *IIA* string theory is an eleven dimensional supergravity theory. Thus it is natural to expect that the spectrum of Bogomol'nyi saturated states in the two theories to be identical in order to test the conjecture. We start with the conjecture [5,7,9] that an eleven dimensional supergravity with an underlying low energy effective fivebrane theory after double dimensional reduction on $K3$ can be identified with a low energy effective heterotic string theory compactified on T^3 . Now, we further compactify the corresponding seven dimensional theories with the metric, three form potential and the gauge fields on a two torus T^2 to obtain a five dimensional theory. We find that the $N = 1$, $D = 11$ supergravity compactified on $K3 \times T^2$ has $N = 2$ supersymmetry in the resulting five dimensions. A similar type of compactification of an eleven dimensional supergravity theory on $K3 \times T^3$ has been discussed in a different context in ref. [40]. At present, there is a strong evidence [41] that a ten dimensional $E_8 \times E_8$ heterotic string is related to an eleven dimensional theory on an orbifold $R^{10} \times S^1/Z_2$, which is the strong coupling limit of the string theory.

With this motivation, we consider the bosonic part of the eleven dimensional $N = 1$ supergravity action [37] with an underlying low-energy membrane world-volume. The massless bosonic field contents are the metric G and a three form potential \mathcal{C}_3 . We write the action as

$$S = \int d^{11}x \sqrt{-G} \left(\mathcal{R}^{(11)} + E_4^2 \right) + \frac{1}{(12)^2} \int \mathcal{C}_3 \wedge E_4 \wedge E_4, \quad (7)$$

where $\mathcal{R}^{(11)}$ is the eleven dimensional scalar curvature and $E_4 = d\mathcal{C}_3$ is the four form anti-symmetric field strength.

Now as discussed in ref. [5,9], the simultaneous dimensional reduction of the eleven dimensional membrane theory (7) on $K3$ gives rise to a three form potential \mathcal{C}_3 , 22 abelian gauge fields $\hat{\mathcal{A}}$ and a metric \mathcal{G} in seven dimensions and the worldvolume reduces to a world-sheet. One can write down the supergravity effective action as

$$S = \int d^7x \sqrt{-\mathcal{G}} e^{\frac{2\hat{\phi}}{3}} \left[R^{(7)} + \frac{1}{3}(\partial\hat{\phi})^2 + \frac{1}{8}Tr(L'\partial K'L'\partial K') - \frac{1}{48}\hat{E}_4^2 \right] \\ - \frac{1}{4} \int d^7x \sqrt{-\mathcal{G}} \hat{F}^{(I')}(L'K'L')_{I'J'} \hat{F}^{(J')} + \frac{1}{(12)^2} \int \mathcal{C}_3 \wedge \hat{F}^{I'} \wedge L_{I'J'} \hat{F}^{J'} \quad (8)$$

where $\hat{\phi}$ is the seven dimensional dilaton, $\hat{F}^{(I')}$ for $I', J' = 1, 2, \dots, 22$ are the gauge field strengths originated from the four form field strength, K' corresponds to the moduli field on $K3$ and $L'_{I'J'}$ is a constant (22×22) matrix defined on $K3$. The four form field strength in seven dimensions is defined as $\hat{E}_4 = d\hat{\mathcal{C}}_3$. Now further compactifying on a two torus T^2 following the prescription in ref. [39], we get

$$S = \int d^5x \sqrt{-\tilde{g}} e^{\frac{2\phi}{3}} \left[R^{(5)} + \frac{1}{3}\tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{8}Tr(\partial_\mu G^{\alpha\beta} \partial^\mu G_{\alpha\beta}) + \frac{1}{8}\tilde{g}^{\mu\nu} Tr(L\partial_\mu K L\partial_\nu K) \right]$$

$$\begin{aligned}
& -\frac{1}{4}F^{(\alpha)}G_{\alpha\beta}F^{(\beta)} - \frac{1}{48}F'_{\mu\nu\rho\sigma}F'^{\mu\nu\rho\sigma} - \frac{1}{12}\tilde{H}_{\mu\nu\rho}^{(\alpha)}\tilde{H}^{\mu\nu\rho}_{(\alpha)} - \frac{1}{4}\tilde{F}_{\mu\nu}^{(\alpha\beta)}G_{\alpha\gamma}G_{\beta\delta}\tilde{F}^{(\gamma\delta)\mu\nu} \\
& -\frac{1}{4}\int d^5x\sqrt{-\tilde{g}}\,G\left[F^{(I)}_{\mu\nu}(LKL)_{IJ}F^{(J)\mu\nu} + 2(LKL)_{IJ}\partial_\mu\chi^I_\alpha\partial^\mu\chi^{J\alpha}\right] \\
& +\frac{1}{4}\int\left[L_{IJ}\,A\wedge F^I\wedge F^J + L_{IJ}\,\tilde{B}\wedge F^I\wedge F^J + \frac{1}{3}L_{IJ}\,C_3\wedge d\chi^I\wedge d\chi^J\right], \quad (9)
\end{aligned}$$

where $I, J, = 1, 2, \dots, 22$ and G is the determinant of $G_{\alpha\beta}$. $(LKL)_{IJ}$ is a 22×22 matrix and is a function of the moduli fields $A^I \equiv \chi^I$. The five dimensional dilaton ϕ is originated from the compactification of the eleven dimensional supergravity theory and is related to the seven dimensional dilaton $\hat{\phi}$ through field redefinitions. The background field configurations contain the Kaluza-Klein gauge fields $\tilde{A}^{(\alpha)}$ for $\alpha = 1, 2$. Along with that there are one form potentials $A^{(\alpha)}$, two form potentials $\tilde{B}^{(\alpha)}$ and a three form potential $C_{\mu\nu\rho}$ which have originated from a single three form potential \hat{C}_3 in seven dimensions. By now it is needless to say that the three form field strengths and the four form one are defined with appropriate gauge Chern-Simon terms and are given by

$$\begin{aligned}
& \tilde{H}_{\mu\nu\rho}^{(\alpha)} = \partial_{[\mu}B^{(\alpha)}_{\nu\rho]} - A^{(\alpha\beta)}_{[\mu}G_{\beta\delta}\tilde{F}_{\nu\rho]}^{(\delta)} \\
& \text{and} \quad F'_{\mu\nu\rho\lambda} = \partial_{[\mu}C_{\nu\rho\lambda]} - F^{(\alpha)}_{[\mu\nu}G_{\alpha\beta}B^{(\beta)}_{\rho\lambda]}. \quad (10)
\end{aligned}$$

There are altogether 26 gauge fields, out of which 22 owe their origin to $K3$ compactification. Of the remaining four, two gauge fields appear from the seven dimensional metric when we compactify on T^2 and other two come from the antisymmetric tensor field. Note that in ref. [42], various dimensionally reduced field strengths obtained from the higher dimensional one have also been discussed in a different context.

IV. POINCARÉ DUALITY

In this section, we apply Poincaré duality on the five dimensional supergravity theory, discussed in section III, to obtain a dual theory. We have calculated the background fields in the five dimensional supergravity theory and find them to be identical in number to that of the low-energy effective heterotic string compactified on a five torus after using the Poincaré duality on certain field strengths. We explicitly show that with a Weyl scaling of the metric and twisting the field strengths, it is possible to identify the $D = 11$ supergravity action compactified on $K3 \times T^2$ with that of heterotic string action on T^5 .

Let us consider the five dimensional action (9) obtained from the eleven dimensional supergravity compactified on $K3 \times T^2$ and rescale the metric as follows :

$$\tilde{g}_{\mu\nu} = e^{-2\Phi}g_{\mu\nu} \quad \text{and} \quad \phi = 3\Phi, \quad (11)$$

where $g_{\mu\nu}$ is the new metric, the Φ is the dilaton field after rescaling and subsequently in the later part of this section, we will identify them with that of the heterotic string effective action in five dimensions. Now, we write down the action from eq.(9) after the rescaling of the metric as in eq.(11),

$$\begin{aligned}
S = \int d^5x \sqrt{-g} e^{-\Phi} & \left[R^{(5)} + g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} g^{\mu\nu} \text{Tr}(L \partial_\mu K L \partial_\nu K) + \frac{1}{4} g^{\mu\nu} \text{Tr}(\eta \partial_\mu M \eta \partial_\nu M) \right. \\
& - \frac{1}{4} e^{2\Phi} F^{(\alpha)}_{\mu\nu} \eta_{\alpha\beta} F^{(\beta)\mu\nu} - \frac{1}{12} e^{4\Phi} \tilde{H}^{(\alpha)}_{\mu\nu\rho} \eta_{\alpha\beta} \tilde{H}^{(\beta)\mu\nu\rho} - \frac{1}{4} e^{2\Phi} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \\
& - \frac{1}{4} F^{(I)}_{\mu\nu} (LKL)_{IJ} F^{(J)\mu\nu} - \frac{1}{2} e^{-2\Phi} (LKL)_{IJ} \partial_\mu \chi^{I,\alpha} \eta_{\alpha\beta} \partial^\mu \chi^{J,\beta} \Big] \\
& + \frac{1}{4} \int \left[L_{IJ} A \wedge F^I \wedge F^J + L_{IJ} B \wedge F^I \wedge F^J \right. \\
& \left. + \frac{1}{3} L_{IJ} C_3 \wedge d\chi^I \wedge d\chi^J + 2\Psi \cdot F^{(\alpha)} \wedge \eta_{\alpha\beta} H^{(\beta)} \right], \tag{12}
\end{aligned}$$

where for simplicity, we have taken the matrix

$$G_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The three form antisymmetric field strength $\tilde{H}^{(\alpha)}$ in eq.(12) is defined with the gauge Chern-Simon term as mentioned in eq.(10). The dual of four form field strength F' is a five dimensional pseudo scalar axion Ψ which can be obtained by using the Poincare duality transformations given as

$$F'^{\mu\nu\rho\sigma} = \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{\sqrt{-g}} e^{-5\Phi} \partial_\lambda \Psi. \tag{13}$$

The two dimensional moduli field M in action (12) is constructed with the five dimensional axion Ψ and the dilaton coupling $e^{-\Phi}$ for consistent counting of the scalar fields parametrizing the moduli field \mathcal{M} of the heterotic string on T^5 . The $SL(2, R)$ parametrization of the moduli field can be written as

$$M = \begin{pmatrix} e^{-2\Phi} & \Psi e^{-2\Phi} \\ \Psi e^{-2\Phi} & \Psi^2 e^{-2\Phi} + e^{2\Phi} \end{pmatrix}, \tag{14}$$

leading to a manifestly $SL(2, R)$ invariant term in the action (12). However, we have not succeeded to write an $SL(2, R)$ invariant form of the full action (12). Now, we use the string-particle Poincare duality on three forms $\tilde{H}^{(\alpha)}$ and a two form field strength \tilde{F} in order to identify the dual action with that of a heterotic string discussed in section III. In passing, we would like to point out an interesting feature of the duality transformation (13) and the construction of the moduli (14). Notice that the last term in (12) is a topological term, which involves the two form $F^{(\alpha)}$, the three form $H^{(\alpha)}$ field strengths ($\alpha = 1, 2$) and Ψ

playing the role of axion (normally denoted by θ in four dimensions). Moreover, it is easy to see that M parametrizes a moduli $SL(2, R)/SO(2)$ similar to a dilaton-axion system in four dimensions.

The dual of the three form $\tilde{H}^{(\alpha)\mu\nu\rho}$ in five dimensions can be written as

$$\tilde{H}^{(\alpha)\mu\nu\rho} = \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{2!\sqrt{-g}} e^{-3\Phi} \hat{F}_{\sigma\lambda}^{(\alpha)} \quad (15)$$

and that of the two form $\tilde{F}^{\mu\nu}$ is

$$\tilde{F}^{\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{3!\sqrt{-g}} e^{-\Phi} \hat{H}_{\rho\sigma\lambda} . \quad (16)$$

Now, we write the $D = 11$ supergravity action compactified on $K3 \times T^2$ in terms of dual fields using eqs.(13),(15) and (16) as

$$\begin{aligned} S = \int d^5x \sqrt{-g} e^{-\Phi} & \left[R^{(5)} + g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} g^{\mu\nu} \text{Tr}(L \partial_\mu K L \partial_\nu K) + \frac{1}{4} g^{\mu\nu} \text{Tr}(\eta \partial_\mu M \eta \partial_\nu M) \right. \\ & - \frac{1}{4} e^{2\Phi} F^{(\alpha)}_{\mu\nu} \eta_{\alpha\beta} F^{(\beta)\mu\nu} - \frac{1}{4} e^{-2\Phi} \hat{F}_{\mu\nu}^{(\alpha)} \eta_{\alpha\beta} \hat{F}^{(\beta)\mu\nu} - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} \\ & - \frac{1}{4} F^{(I)}_{\mu\nu} (LKL)_{IJ} F^{(J)\mu\nu} - \frac{1}{2} e^{-2\Phi} (LKL)_{IJ} \partial_\mu \chi^{I,\alpha} \eta_{\alpha\beta} \partial^\mu \chi^{J,\beta} \\ & + \frac{1}{2} e^{-2\Phi} \Psi \cdot \hat{F}_{\mu\nu}^{(\alpha)} \eta_{\alpha\beta} F^{(\beta)\mu\nu} - \frac{1}{2} e^{-4\Phi} \chi^I \cdot L_{IJ} \partial^\mu \chi^J \partial_\mu \Psi - \frac{1}{2} e^{-2\Phi} \chi^I \cdot L_{IJ} F^{J\mu\nu} \hat{F}_{\mu\nu} \Big] \\ & + \frac{1}{4} \int \left[L_{IJ} A \wedge F^I \wedge F^J + A^{(\alpha)} \eta_{\alpha\beta} \wedge F^{(\beta)} \wedge \hat{F} + A^{(\alpha)} \wedge \eta_{\alpha\beta} F^{(\beta)} \wedge \hat{F} \right] \end{aligned} \quad (17)$$

There are altogether 26 gauge fields $\mathcal{A}_\mu^{(a)}$ for $a = 1, 2, \dots, 26$, a single antisymmetric potential $\hat{B}_{\mu\nu}$, whose field strength is $\hat{H}_{\mu\nu\rho}$ and a set of moduli fields χ 's in the action (17). Suitably incorporating the $K3$ moduli fields K , $SL(2, R)$ invariant moduli M and the $(LKL)_{IJ}$ from $K3 \times T^2$, arising out of the compactification on $K3 \times T^2$, we define \tilde{M}_{ij} and \tilde{C}_{ab} , to rewrite the above rescaled action (17) in terms of the redefined moduli fields $\tilde{\mathcal{M}}$. In terms of Einstein metric some of the terms in the action (17) can be shown to be manifestly $SL(2, R)$ invariant. Note that $\tilde{\mathcal{M}}$ represents a set of 105 scalars along with the dilaton and can be identified with that of the heterotic string on a five torus discussed in the preceeding section. The dilaton and 57 of scalars parametrize the moduli K coming from the $K3$ compactification of eleven dimensional supergravity. The remaining 48 of the scalars arise from the T^2 compactification of the seven dimensional supergravity theory, *e.g.* 44 of them from the gauge field, 3 from metric and the remaining one is Poincare dual of two form potential. The two form potential $\hat{B}_{\mu\nu}$ is obtained from the Poincare dual of the spin one field, which has its origin in the three form potential \hat{C}_3 . Similarly for the 26 gauge fields in five dimensional supergravity theory; 22 of them come from the gauge fields, two each from the metric and the two form potential

in seven dimensions. We write the respective gauge fields $A^{(I)}, A^{(\alpha)}$ and $\hat{A}^{(\alpha)}$ in a multiplet as

$$\mathcal{A}_{\mu\nu}^{(a)} = \begin{pmatrix} A^{(I)} \\ A_{\mu\nu}^{(\alpha)} \\ \hat{A}^{(\alpha)} \end{pmatrix} \quad (18)$$

and write the rescaled supergravity action (17) as

$$S = \int d^5x \sqrt{-g} e^{-\Phi} \left[R^{(5)} + g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \tilde{M}_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - \frac{1}{12} g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'} \hat{H}_{\mu\nu\rho} \hat{H}_{\mu'\nu'\rho'} \right. \\ \left. - \frac{1}{4} g^{\mu\mu'} g^{\nu\nu'} \mathcal{F}_{\mu\nu}^{(a)} \tilde{C}_{ab} \mathcal{F}_{\mu'\nu'}^{(b)} \right] + \frac{1}{4} \int \mathcal{L}_{ab} A^{(a)} \wedge \mathcal{F}^{(b)} \wedge \hat{F}, \quad (19)$$

where

$$\mathcal{L} = \begin{pmatrix} -L_{IJ} & 0 & 0 \\ 0 & \eta_{\alpha\beta} & 0 \\ 0 & 0 & \eta_{\alpha\beta} \end{pmatrix}, \quad (20)$$

$$\tilde{M}_{ij} = \begin{pmatrix} N_{i'j'} & 0 & 0 \\ 0 & \frac{e^{-2\Phi}}{2} & \frac{e^{-4\Phi}}{4} L_{IJ} \chi^J \\ 0 & \frac{e^{-4\Phi}}{4} L_{IJ} \chi^I & \frac{e^{-2\Phi}}{2} [(LKL)_{IJ} \times \eta_{\alpha\beta}] \end{pmatrix} \quad (21)$$

and

$$\tilde{C}_{ab} = \begin{pmatrix} (LKL)_{IJ} & 0 & e^{-2\Phi} L_{IJ} \chi^J \\ 0 & e^{2\Phi} \eta_{\alpha\beta} & -\Psi e^{-2\Phi} \eta_{\alpha\beta} \\ e^{-2\Phi} L_{IJ} \chi^I & -\Psi e^{-2\Phi} \eta_{\alpha\beta} & e^{-2\Phi} \eta_{\alpha\beta} \end{pmatrix}, \quad (22)$$

where $N_{i'j'}$ is a matrix and is a function of the moduli fields on $K3 \times T^2$. The scalar fields φ^i of the moduli \tilde{M} can be identified with those of heterotic string moduli M as in eq.(6).

The action in eq.(19) is obtained by compactifying the $D = 11$ supergravity theory with an underlying membrane effective action on $K3 \times T^2$. The field content in the massless sector is found to be equivalent to that of the heterotic string effective action in five dimensions. Thus a rescaling (11) of the five dimensional supergravity metric along with the Poincare duality transformations (13), (15) and (16), correspond to a toroidally compactified heterotic string theory with $N = 2$ space-time supersymmetry. Thus the strong coupling dynamics of heterotic string theory in five dimensions is identical to the $K3 \times T^2$ compactified $D = 11$ supergravity theory.

V. STRONG-WEAK COUPLING DUALITY

In this section, we discuss the strong-weak coupling string-particle duality of an eleven dimensional supergravity theory compactified on $K3 \times T^2$, whose action is given in eq.(9). As discussed in section I, the string-string duality relates a strongly coupled ten dimensional low energy heterotic string theory compactified on T^4 to a weakly coupled ten dimensional Type *IIA* string compactified on $K3$ and viceversa. As a result a Type *IIA* theory on $K3$ gets enhanced gauge symmetries. Thus it is natural to test explicitly the strong-weak coupling limit of the compactified eleven dimensional supergravity theory on $K3 \times T^2$ with that of the Type *IIA* string on $K3 \times S^1$ in order to arrive at a set of relations between various theories.

Now, we start by Weyl scaling the metric \tilde{g} and redefining the dilaton field as

$$\tilde{g}_{\mu\nu} = e^{\frac{2\Phi'}{3}} g'_{\mu\nu}, \quad \phi = -3\Phi', \quad (23)$$

where the $g'_{\mu\nu}$ and Φ' denote the Type *IIA* string metric and the dilaton respectively. We rewrite the action (9) in terms of these new fields as

$$\begin{aligned} S = \int d^5x \sqrt{-g'} e^{-\Phi'} & \left[R^{(5)} + g'^{\mu\nu} \partial_\mu \Phi' \partial_\nu \Phi' + \frac{1}{4} \text{Tr}(\partial_\mu G^{\alpha\beta} \partial^\mu G_{\alpha\beta}) + \frac{1}{8} g'^{\mu\nu} \text{Tr}(L \partial_\mu K L \partial_\nu K) \right. \\ & + \frac{1}{4} g'^{\mu\nu} \text{Tr}(\partial_\mu M^{-1} \partial_\nu M) - \frac{1}{4} e^{\frac{-2\Phi'}{3}} F^{(\alpha)} G_{\alpha\beta} F^{(\beta)} \\ & - \frac{1}{12} e^{\frac{-4\Phi'}{3}} \tilde{H}_{\mu\nu\rho}^{(\alpha)} \tilde{H}_{(\alpha)}^{\mu\nu\rho} - \frac{1}{4} e^{\frac{-2\Phi'}{3}} \tilde{F}_{\mu\nu}^{(\alpha\beta)} G_{\alpha\gamma} G_{\beta\delta} \tilde{F}^{(\gamma\delta)\mu\nu} \left. \right] \\ & - \frac{1}{4} \int d^5x \sqrt{-g'} G e^{\frac{4\Phi'}{3}} \left[F^{(I)}{}_{\mu\nu} (LKL)_{IJ} F^{(J)\mu\nu} + 2e^{2\Phi'} (LKL)_{IJ} \partial_\mu \chi^I{}_\alpha \partial^\mu \chi^{J\alpha} \right] \\ & + \frac{1}{4} \int \left[L_{IJ} A \wedge F^{(I)} \wedge F^{(J)} + L_{IJ} B \wedge F^{(I)} \wedge F^{(J)} + \frac{1}{3} L_{IJ} C_3 \wedge d\chi^I \wedge d\chi^J \right] \quad (24) \end{aligned}$$

In this case, the Poincare dual of the four form field strength besides the gauge Chern-Simon term is given by

$$F'^{\mu\nu\rho\sigma} = \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{\sqrt{-g'}} e^{3\Phi'} \partial_\lambda \Psi' \quad (25)$$

We have used the expression in eq.(25) to write the above action (24). The dual of three form \tilde{H} and the two form F are given by

$$\begin{aligned} \tilde{H}^{\mu\nu\rho} &= \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{2! \sqrt{-g'}} e^{\frac{7\Phi'}{3}} \hat{F}_{\sigma\lambda} \\ \text{and} \quad F^{\mu\nu} &= \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{3! \sqrt{-g'}} e^{\frac{\Phi'}{3}} \hat{H}_{\rho\sigma\lambda}, \end{aligned} \quad (26)$$

where the three form field strength $\hat{H}_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$. Now, we write the dual form of the action (24) using eq.(25)-(26) as

$$\begin{aligned}
S = \int d^5x \sqrt{-g'} e^{-\Phi'} & \left[R^{(5)} + g'^{\mu\nu} \partial_\mu \Phi' \partial_\nu \Phi' + \frac{1}{8} g'^{\mu\nu} \text{Tr}(\partial_\mu K^{-1} \partial_\nu K) + \frac{1}{4} g'^{\mu\nu} \text{Tr}(\partial_\mu M^{-1} \partial_\nu M) \right. \\
& - \frac{1}{4} e^{\frac{-2\Phi'}{3}} F^{(\alpha)} \eta_{\alpha\beta} F^{(\beta)} + \frac{1}{2} e^{\frac{10\Phi'}{3}} \Psi' \cdot F^{(\alpha)} \eta_{\alpha\beta} \hat{F}^{(\beta)} - \frac{1}{4} e^{\frac{10\Phi'}{3}} \hat{F}_{\mu\nu}^{(\alpha)} \eta_{\alpha\beta} \hat{F}^{(\beta)\mu\nu} \\
& - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} - \frac{1}{4} e^{\frac{4\Phi'}{3}} F^{(I)}{}_{\mu\nu} (LKL)_{IJ} F^{(J)\mu\nu} - \frac{1}{2} e^{2\Phi'} [(LKL)_{IJ} \times \eta_{\alpha\beta}] \partial_\mu \chi^{I,\alpha} \partial^\mu \chi^{J,\beta} \\
& + \frac{1}{2} \Psi' \cdot \partial_\mu \chi^I L_{IJ} \partial^\mu \chi^J - \frac{1}{2} e^{\frac{10\Phi'}{3}} \chi^I \cdot L_{IJ} F^{J\mu\nu} \hat{F}_{\mu\nu} \Big] \\
& + \frac{1}{4} \int [L_{IJ} A \wedge F^{(I)} \wedge F^{(J)} + \eta_{\alpha\beta} A^{(\alpha)} \wedge F^{(\beta)} \wedge \hat{F}_1 + \eta_{\alpha\beta} A^{(\alpha)} \wedge F^{(\beta)} \wedge \hat{F}_2] , \quad (27)
\end{aligned}$$

where we have considered $G_{\alpha\beta} = \eta_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) as in section IV. In order to identify the action (27) with that of a Type *IIA* string compactified on $K3 \times S^1$, we redefine the 26 of the gauge fields, $A^{(I)}$, $A^{(\alpha)}$ and $\hat{A}^{(\beta)}$ in a gauge field multiplet $\mathcal{A}^{(a)}$ for $a = 1, 2, \dots, 26$. As a result, we identify these gauge fields with that in the Ramond-Ramond sector of a dimensionally reduced Type *IIA* string. Identifying the twisted moduli fields on the $K3 \times T^2$ with the matrix $N_{i'j'}$, one can arrive at the five dimensional string effective action given by

$$\begin{aligned}
S = \int d^5x \sqrt{-g'} e^{-\Phi'} & \left[R^{(5)} + g'^{\mu\nu} \partial_\mu \Phi' \partial_\nu \Phi' - \mathcal{N}_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - \frac{1}{12} g'^{\mu\mu'} g'^{\nu\nu'} g'^{\rho\rho'} H'_{\mu\nu\rho} H'_{\mu'\nu'\rho'} \right] \\
& - \frac{1}{4} \int d^5x \sqrt{-g'} F^{(a)} \mathcal{D}_{ab} F^{(b)} - \frac{1}{4} \int L_{ab} A \wedge F^{(a)} \wedge F^{(b)} , \quad (28)
\end{aligned}$$

where

$$\mathcal{D}_{ab} = \begin{pmatrix} e^{\frac{-\phi'}{3}} (LKL)_{IJ} & 0 & e^{\frac{10\phi'}{3}} L_{IJ} \chi^J \\ 0 & e^{\frac{-2\phi'}{3}} \eta_{\alpha\beta} & -\Psi' e^{\frac{10\phi'}{3}} \eta_{\alpha\beta} \\ e^{\frac{10\phi'}{3}} L_{IJ} \chi^I & -\Psi' e^{\frac{10\phi'}{3}} \eta_{\alpha\beta} & e^{\frac{10\phi'}{3}} \eta_{\alpha\beta} \end{pmatrix} \quad (29)$$

and

$$\mathcal{N}_{ij} = \begin{pmatrix} N_{i'j'} & 0 & 0 & 0 \\ 0 & \frac{e^{4\phi'}}{2} & 0 & 0 \\ 0 & 0 & L_{IJ} \Psi' & 0 \\ 0 & 0 & 0 & \frac{1}{2} e^{2\phi'} [(LKL)_{IJ} \times \eta_{\alpha\beta}] \end{pmatrix} \quad (30)$$

for $a, b = 1, 2, \dots, 26$ and $i, j = 1, 2, \dots, 105$. The action in eq.(28) corresponds to a ten dimensional Type *IIA* string compactified on $K3 \times S^1$. The transformations (23) along with (25) and (26) acting on the supergravity action (24) leads to a Type *IIA* dimensionally reduced string action in five dimensions with the gauge fields in the Ramond-Ramond sector. In fact, the five dimensional Type *IIA* string effective action is obtained from the corresponding supergravity action by applying the strong-weak coupling duality as the dilaton of one theory is negative of other (23). At this point note that the dilaton in the five dimensional

Type *IIA* string theory has its origin in the toroidal compactification of the supergravity theory whereas the one in heterotic string has its origin in the $K3$ moduli fields.

Thus we have explicitly shown the relation between a dimensionally reduced supergravity theory with that of heterotic string and the Type *IIA* string theory in five dimensions supporting the evidences in ten dimensions [41]. Since the massless states of a Type *IIA* string on $K3 \times S^1$ is identical to that of a Type *IIB* on $K3 \times S^1$, we state a chain of interconnections in five dimensions between various string theories and the supergravity theory supporting the conjectures in seven and six dimensions. Furthermore by analyzing the coset $O(5, 21)/[O(5) \times O(21)]$ representing the moduli fields of a Type *IIB* string compactified on $K3$ [5], which is indeed similar to the case of Type *IIB* compactified on $K3 \times S^1$, one can find the identical massless states in the five dimensional supergravity theory under discussions. This results in identifying the five dimensional supergravity theory with that of a Type *IIB* string in six dimensions which has been given an interpretation of large radius (S^1) limit of the five dimensional one. We conclude this section by stating that in five dimensions, the heterotic, Type *IIA* and Type *IIB* string theories are different phases of the dimensionally reduced eleven dimensional supergravity theory.

VI. CHARGED SOLUTIONS OF HETEROTIC STRING EFFECTIVE ACTION

In support of the dualities in five dimensions as discussed in the previous sections, we analyze the classical solutions of the heterotic string effective action and the dual models in this section. We present magnetically charged solitonic solution of the low energy heterotic string effective action in five dimensions. The topological (or Noether) charge of this solution is defined on S^3 corresponding to the two form antisymmetric potential. We extend our analysis for nonsingular point like string solutions and present a spherically symmetric one of the string-particle dual effective action with magnetic charge defined on a two sphere S^2 . By applying the five dimensional Poincare duality discussed in section IV, we interpret them as solutions corresponding to the supergravity theory with an underlying low-energy membrane theory. As a consequence the BPS solitonic solution of heterotic string theory remains a solitonic one of the corresponding supergravity theory and viceversa. However the geometry of the solution changes from S^3 to S^2 with magnetic charge due to gauge field instead of the two form antisymmetric potential. Furthermore, under the strong-weak coupling duality, as discussed in the previous section V, the Noether and topological charges get interchanged [6,7] and the singular solutions go over to the nonsingular ones and viceversa. We argue that these solutions describing the geometry with different topologies are in agreement with the string-particle duality in five dimensions. The interconnections between different theories

in five dimensions admit the classical solutions of heterotic string theory to be that of supergravity theory compactified on $K3 \times T^2$ and Type *IIA* theory on $K3 \times S^1$.

Let us start with the ansatz for the classical solution satisfying the equations of motion of the heterotic string effective action (4). We write down the classical background configurations in five dimensions by winding of the solitonic string solution of the corresponding six dimensional one described by Sen [6].

The string metric defining the invariant distance

$$ds^2 = -dt^2 + e^\Phi \left(dr^2 + r^2 d\Omega_3^2 \right), \quad (31)$$

the dilaton Φ defining the string coupling

$$e^\Phi = 1 \pm \frac{Q}{r^2}, \quad (32)$$

the three form field strength

$$H_{\mu\nu\rho} = \pm \epsilon_{\mu\nu\rho\lambda} \partial^\lambda \Phi, \quad (33)$$

the gauge fields

$$A_\mu = 0 \quad \text{and the moduli fields} \quad \varphi^i = \text{const.}, \quad (34)$$

where $d\Omega_3^2$ is the $SO(4)$ invariant metric on a three sphere S^3 . ‘ Q ’ is the topological charge carried by the solitonic string states corresponding to the antisymmetric tensor field $B_{\mu\nu}$ in contrast to that of A_μ which is taken to be zero in this case. The volume form is defined on a three sphere S^3 with $\mu, \nu, \rho, \lambda = r, \theta, \phi, \psi$. The solution in eq.(31)-(34) satisfy the background field equations of motion corresponding to the heterotic string effective action. The Ricci scalar curvature is calculated for the above backgrounds and found to be nonsingular at $r = 0$ and is asymptotically flat. This magnetically charged solution can be interpreted as a BPS “ H -monopole” like solution in five dimensions which after compactification of the three sphere to a two sphere becomes a H -monopole solution in four dimensions. The magnetic charge corresponding to the two form potential in the above eq.(31)-(34) can be calculated by using the general expression given by

$$Q \equiv \int_{S^3} H.$$

Now we consider a dual picture in the framework of string-particle dualities, namely the Poincare duality and the strong-weak coupling duality, by considering the classical charged solution of the low-energy heterotic string effective action. We interpret various duals of

the heterotic string effective theory (4) by replacing the two form potential B with that of a spin one gauge field \tilde{A} whose field strength \tilde{F} is defined in eq.(5). In this framework (6) we seek for spherically symmetric solitonic solution and discuss their relation with the one in eqs.(31)-(34). The backgrounds satisfying the equations of motion derived from the action in eq.(6) are :

the string metric defining an invariant distance

$$ds^2 = -dt^2 + dx^2 + e^\Phi (dr^2 + r^2 d\Omega_2^2) , \quad (35)$$

the dilaton Φ with a string coupling

$$e^\Phi = 1 \pm \frac{Q'}{r} , \quad (36)$$

the only gauge field strength

$$\tilde{F}_{\mu\nu} = \pm\sqrt{2} \epsilon_{\mu\nu\lambda} \partial^\lambda \Phi , \quad (37)$$

the 26 components gauge field multiplet

$$A_\mu^a = 0 \quad \text{and the moduli fields} \quad \varphi^i = \text{const.} \quad (38)$$

The Q' is the topological charge of the solitonic string states corresponding to the gauge field A_μ . The volume form is defined on a two sphere S^2 with $\mu, \nu, \rho = r, \theta, \phi$. The only nonvanishing gauge field satisfying the eq.(37) is given by

$$\tilde{A}_\phi = \sqrt{2}Q'(1 - \cos\theta) .$$

We have explicitly verified that the above set of metric, dilaton, gauge fields and the moduli fields satisfy the background field equations of motion obtained from the dual version of heterotic string effective action (6) with magnetic charge Q' defined on a two sphere S^2 . This represents a magnetically charged BPS solution with charge defined on a two sphere S^2 of radius r . In the asymptotic limit, $r \rightarrow \infty$, the scalar curvature vanishes and the solution becomes flat. The solution (35)-(38) is translationally invariant in x - direction and represents a spherically symmetric magnetically charged regular geometry with charge defined on S^2 of radius r . The solution can be interpreted as a monopole solution in five dimensions and by wrapping the x -direction in eq.(35) along the string, the solution can be identified with the *BPS* monopole solution in four dimensions. The conserved magnetic charge can also be calculated, with a similar expression as in the previous section, using the dual field strengths and can be written as

$$Q' \equiv \int_{S^2} \tilde{F} .$$

The charged solution (35)-(38) can be interpreted as a consequence of string-particle duality which in turn relates the heterotic string action to the supergravity, Type *IIA* and *IIB* string theories. The solutions (35)-(38) can be argued to obtain from that of in eqs.(32)-(34) by applying Poincare duality which transforms a two form potential to a spin one field. At this point we would like to recall that a 28 parameter charged classical string solution with nontrivial moduli field is discussed in ref. [43]. The consequence of a series of connections relating various phases of the eleven dimensional supergravity compactified on a $K3 \times T^2$ to heterotic string compactified on T^5 and that of Type *IIA* string on $K3 \times S^1$ in various limits suggests that the singular solution in a fundamental theory may be interpreted as the soliton solution of the dual theory and viceversa.

Note that dual theories admit magnetically charged solutions which are due to the two form potential and the gauge fields respectively. Since the Poincare duality relates a two form potential with a spin one gauge field, we interpret these two charged solutions as dual to each other which interchanges the geometries namely from a three sphere S^3 to a two sphere S^2 . However in absence of charges these solutions become trivial. Further applying strong-weak coupling duality on the heterotic string effective action (4) which is identified with a supergravity action, one can arrive at a Type *IIA* string effective action. As a result, along with a change in geometries of the magnetically charged solitonic solutions becomes a singular one and the Noether and topological charges get interchanged.

VII. DISCUSSION

In this paper, we have demonstrated the explicit interconnections between the $N = 1$, $D = 11$ supergravity compactified on $K3 \times T^2$ with that of a $D = 10$ heterotic string theory on a specific T^5 and $D = 10$ Type *IIA*, Type *IIB* string theories on $K3 \times S^1$. We have utilized the string-membrane duality conjecture in seven dimensions along with a more stronger one, namely string-string duality in six dimensions in order to provide an evidence for the string-particle duality in five dimensions. We note that the speciality of five dimensions permit us to construct an axion which in fact is a Poincare dual to the four form field strength and appears as a topological term in the theory.

We have also presented magnetically charged classical solutions in five dimensions with charge defined on a three sphere S^3 and a two sphere S^2 respectively. These backgrounds are analyzed and we interpret them as BPS monopole like solutions analogous to that in four dimensions. By our construction, we have started with the solutions of different topologies which are realized as a consequence of the string-particle dualities in the effective

action and manifested in the guise of Poincare duality and the strong-weak coupling one. These magnetically charged solutions, in fact can be interpreted as that of the dimensionally reduced supergravity, Type *IIA* and *IIB* string theories through the string-particle dualities discussed. To be specific, we have presented two solitonic solutions of the heterotic string effective action on a five torus with the trivial moduli fields. It is shown that H -monopole solutions with charge corresponding to the two form potential (defined on S^3) and that with respect to the gauge fields (defined on S^2) are obtained from the dual pair of heterotic string theories. In the analysis, we have used the fact that the two form potential is dual to gauge field and viceversa which is the origin of string-particle duality in the present context. We note that the five dimensional duality is accompanied by a change of topology from S^3 to S^2 and viceversa in contrast to the four dimensional particle-particle and six dimensional string-string duality where the geometries remain unchanged. However in the present context of nonperturbative duality symmetry, the string-string duality is in much stronger footing than the string-particle duality in five dimensions and the string-membrane duality in seven dimensions. In passing, we recall that the possible puzzle analogous to that of the H -monopole problem in four dimensions [30] still remains unanswered in this context. The string dynamics in five dimensions has also been analyzed recently in ref. [44] in a different context.

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